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Analysis of the neutralino system in three–body leptonic decays of neutralinos

S.Y. Choi[1], B.C. Chung[2], J. Kalinowski[3,a], Y.G. Kim[4], K. Rolbiecki[3]

[1] Department of Physics, Chonbuk National University, Jeonju 561-756, Korea
[2] Department of Physics, KAIST, Daejon 305–017, Korea
[3] Institute of Theor. Phys., Warsaw University, Hoza 69, 00681 Warsaw, Poland
[4] Department of Physics, Korea University, Seoul 136–701, Korea

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Abstract. Neutralinos $\tilde{\chi}^0$ in supersymmetric theories, the spin–1/2 Majorana–type superpartners of the U(1) and SU(2) neutral electroweak gauge bosons and SU(2) neutral Higgs bosons, are expected to be among light supersymmetric particles so that they can be produced copiously via direct pair production and/or from cascade decays of other sparticles such as sleptons at the planned Large Hadron Collider and the prospective International Linear Collider. Considering the prospects of having both highly polarized neutralinos and possibility of reconstructing their decay rest frames, we provide a systematic investigation of the three–body leptonic decays of the neutralinos in the minimal supersymmetric standard model and demonstrate alternative ways for probing the Majorana nature of the neutralinos and CP violation in the neutralino system.

1 Introduction

The search for supersymmetry (SUSY) is one of the main goals at present and future colliders since SUSY is generally accepted as one of the most promising concepts for physics beyond the standard model (SM) [1]. All SUSY theories contain neutralinos, the spin–1/2 Majorana superpartners of neutral gauge bosons and Higgs bosons, that are expected to be among the light supersymmetric particles that can be produced copiously at future high energy colliders.

Once neutralino candidates have been detected at high energy colliders such as the Large Hadron Collider (LHC) [2] and the International Linear Collider (ILC) [3], it is of great importance to verify that the observed states are indeed the spin–1/2 superpartners of the neutral SM gauge and Higgs bosons. For that purpose, it will be crucial to measure their quantum numbers and to confirm that they are indeed Majorana fermions [4–7]. Moreover, masses, mixings, couplings and CP violating phases must be measured in a model–independent way [8–10] to reconstruct the fundamental SUSY parameters and to verify the SUSY relations at the electroweak scale, leading to a reliable extrapolation to the grand unification scale or the Planck scale [11].

In this report we focus on probing the Majorana nature and CP properties of neutralinos in the minimal supersymmetric standard model (MSSM) through the *charge (C) self–conjugate* three–body decays of polarized neutralinos into the lightest neutralino $\tilde{\chi}^0_1$ and a lepton pair $\ell^+\ell^-$:

$$\tilde{\chi}^0_i \to \tilde{\chi}^0_1\, \ell^+\, \ell^- \tag{1}$$

with $\ell = e$ or μ whose four–momenta can be measured with great precision. In particular, the decays of the second lightest neutralino $\tilde{\chi}^0_2$ will be studied in more detail since it is expected in most supersymmetric scenarios [12] that the three–body decay mode has a significant branching fraction only for the second lightest neutralino, while the heavier neutralinos decay mainly through two–body decays.

Thorough analysis of the CP properties and Majorana nature of neutralinos produced in pairs in e^+e^- annihilation has been performed in Refs. [5] and [9] [CP asymmetries in neutralino production with two-body decays have been investigated in Ref. [13]]. The spin–1/2 neutralinos $\tilde{\chi}^0_i$ are produced polarized with the degree of polarization depending on their production mechanism and polarization of the colliding beams. Because in general the momenta of final particles do depend on the neutralino polarization, full account of spin correlations between production and decay processes is necessary [5, 9].

Since the verification of Majorana character of neutralinos and their CP properties are of fundamental importance, they have to be scrutinized in all possible ways. Here we consider neutralinos $\tilde{\chi}^0_2$ which themselves are decay products of scalar particles, *i.e.* sleptons. In such a case their subsequent decay can be analyzed *independently* of the production mechanism making the theoretical treat-

[a] e-mail: kalino@fuw.edu.pl

ment greatly simplified and the interpretation of various observables more transparent.

As noted explicitly in a recent work [14], neutralinos $\tilde{\chi}^0_2$ produced in $\tilde{e}^\pm_L$ decays are 100% polarized, having negative helicity in $\tilde{e}^-_L \to e^-\tilde{\chi}^0_2$ and positive helicity in $\tilde{e}^+_L \to e^+\tilde{\chi}^0_2$. Furthermore, it is possible to reconstruct the rest frame of the neutralino $\tilde{\chi}^0_2$ in a few specific cascade processes, for example, in the process

$$e^+e^- \to \tilde{e}^+_L\tilde{e}^-_L \to e^+\tilde{\chi}^0_1 e^-\tilde{\chi}^0_2 \quad (2)$$

followed by the three–body decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\mu^+\mu^-$ as shown in Refs. [15] and [16]. It is the purpose of this paper to show that such a perfect neutralino polarization combined with the study of angular correlations in the neutralino rest frame can provide us with alternative ways for probing the Majorana nature of the neutralinos[1] and CP violation in the neutralino system.

Keeping in mind the above aspects, we provide in the present work a systematic analysis of the neutralino decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$ in *its decay rest frame* for extracting all the physical implications due to the Majorana nature as well as CP violation [5, 9, 15, 18, 19] in the neutralino system of the MSSM assuming 100% neutralino polarization. Through the present work it is assumed that the neutralino masses have already been measured with great precision [20]. On the other hand, the efficiency of reconstructing the $\tilde{\chi}^0_2$ polarization as well as its rest frame depends not only on the cascade processes under consideration and on the values of relevant SUSY parameters, but also on details of experimental setup. Experimental simulations with realistic reconstruction efficiencies and with background processes included, however, are beyond the scope of the semi-theoretical treatment in the present paper. Nevertheless, we hope that our findings are interesting enough to motivate realistic simulations.

The layout of the paper is as follows. In Sect. 2, we briefly recall the mixing formalism for the neutral gauginos and higgsinos in the CP non–invariant theories with complex phases. In Sect. 3 the leptonic decays $\tilde{\chi}^0_i \to \tilde{\chi}^0_1\ell^+\ell^-$ of a polarized neutralino $\tilde{\chi}^0_i$ $(i = 2, 3, 4)$ are described in terms of quartic charges and Mandelstam kinematic variables, the decay distribution in terms of two lepton energies and three angles in the rest frame of the decaying neutralino is discussed. The consequences of the CP and CP$\tilde{\text{T}}$ invariance on the polarized decay distributions are explained and new relations among the decay amplitudes, *unique* for Majorana particles, are derived. We illustrate in Sect. 4 how to probe the Majorana character and CP violation of the neutralino system through detailed analytical and numerical investigation of various observables in the rest frame of the decaying neutralino $\tilde{\chi}^0_2$: CP–even lepton energy/angular distributions, lepton invariant mass and opening angle distributions, and a CP–odd triple product of the neutralino spin vector and two lepton momenta. Finally, we summarize our findings and conclude in Sect. 5.

[1] A clear independent evidence of the Majorana character of the neutralinos can be provided by an experimental identification of the selectron pair production in e^-e^- collisions, which occur only via t– and u–channel neutralino exchange [15, 17].

2 Neutralino mixing

In the MSSM, the four neutralinos $\tilde{\chi}^0_i$ $(i = 1, 2, 3, 4)$ are mixtures of the neutral U(1) and SU(2) gauginos, $\tilde{B}$ and $\tilde{W}^3$, and the SU(2) higgsinos, $\tilde{H}^0_1$ and $\tilde{H}^0_2$. The neutralino mass matrix in the $(\tilde{B}, \tilde{W}^3, \tilde{H}^0_1, \tilde{H}^0_2)$ basis,

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix} \quad (3)$$

is built up by the fundamental SUSY parameters: the U(1) and SU(2) gaugino masses M_1 and M_2, the higgsino mass parameter μ, and the ratio $\tan\beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields which break the electroweak symmetry. The existence of CP–violating phases in supersymmetric theories in general induces electric dipole moments (EDM). The current experimental bounds on the EDM's can be exploited to derive indirect limits on the parameter space [21].

By reparametrization of the fields, M_2 can be taken real and positive without loss of generality so that the two remaining non–trivial phases, which are reparametrization–invariant, may be attributed to M_1 and μ:

$$M_1 = |M_1|\,\mathrm{e}^{i\Phi_1}, \quad \mu = |\mu|\,\mathrm{e}^{i\Phi_\mu} \;\; (0 \le \Phi_1, \Phi_\mu < 2\pi) \quad (4)$$

Since the matrix $\mathcal{M}$ is symmetric, one unitary matrix N is sufficient to rotate the gauge eigenstate basis $(\tilde{B}, \tilde{W}^3, \tilde{H}^0_1, \tilde{H}^0_2)$ to the mass eigenstate basis of the Majorana fields $\tilde{\chi}^0_i$

$$\mathcal{M}_{\mathrm{diag}} = N^*\mathcal{M}N^\dagger \quad (5)$$

The mass eigenvalues m_i $(i = 1, 2, 3, 4)$ in $\mathcal{M}_{\mathrm{diag}}$ can be chosen real and positive by a suitable definition of the unitary matrix N.

The most general 4×4 unitary matrix N can be parameterized by six angles and ten phases. It is convenient to factorize the matrix N into a diagonal Majorana–type $\mathbf{M}$ and a Dirac–type $\mathbf{D}$ component in the following way:

$$N = \mathbf{MD} \quad (6)$$

The matrix $\mathbf{D}$ can be written as a sequence of 6 independent two–dimensional rotations parameterized in terms of 6 angles and 6 phases. The diagonal matrix $\mathbf{M} = \mathrm{diag}\left\{\mathrm{e}^{i\alpha_1}, \mathrm{e}^{i\alpha_2}, \mathrm{e}^{i\alpha_3}, \mathrm{e}^{i\alpha_4}\right\}$ is given in terms of Majorana phases α_i $(0 \le \alpha_i < \pi)$ [8]. One overall Majorana phase is non-physical and, for example, α_1 may be chosen to vanish.

Due to the Majorana nature of the neutralinos, all nine phases of the mixing matrix N are fixed by underlying SUSY parameters, and they cannot be removed by rephasing the fields. CP is conserved if all the Dirac phases are 0 mod π and the Majorana phases α_i 0 mod $\pi/2$. [Majorana phases $\alpha_i = \pm\pi/2$ describe different CP parities of the neutralino states.] As a consequence, all the matrix elements

$N_{i\alpha}$ are purely real or purely imaginary in the CP invariant case.

3 Three–body leptonic neutralino decays

3.1 Neutralino decay amplitude

The diagrams contributing to the three–body leptonic decay process $\tilde{\chi}^0_i \to \tilde{\chi}^0_1 \ell^+\ell^-$ are shown in Fig. 1. Here, the exchange of the neutral Higgs boson [replacing the Z boson] is neglected since the couplings to the first and second generation SM leptons, $\ell = e$ and μ, are very small. In this case, all the components of the decay matrix elements are of the (vector–current)×(vector–current) form which, after a simple Fierz transformation of the slepton–exchange parts, may be written for the lepton final states as

$$D\left(\tilde{\chi}^0_i \to \tilde{\chi}^0_1 \ell^+\ell^-\right) = \frac{e^2}{m^2_{\tilde{\chi}^0_i}} D_{\alpha\beta}\left[\bar{u}(\tilde{\chi}^0_1)\gamma^\mu P_\alpha u(\tilde{\chi}^0_i)\right]\left[\bar{u}(\ell^-)\gamma_\mu P_\beta v(\ell^+)\right] \quad (7)$$

with the generalized bilinear charges $D_{\alpha\beta}$ $(\alpha,\beta = L,R)$ for the decay amplitudes:

$$\begin{aligned}
D_{LL} &= +\frac{D_Z}{s^2_W c^2_W}(s^2_W - \frac{1}{2})\mathcal{Z}_{1i} - D_{uL}\, g_{L1i}\\
D_{LR} &= +\frac{D_Z}{c^2_W}\mathcal{Z}_{1i} + D_{tR}\, g_{R1i}\\
D_{RL} &= -\frac{D_Z}{s^2_W c^2_W}(s^2_W - \frac{1}{2})\mathcal{Z}^*_{1i} + D_{tL}\, g^*_{L1i}\\
D_{RR} &= -\frac{D_Z}{c^2_W}\mathcal{Z}^*_{1i} - D_{uR}\, g^*_{R1i}
\end{aligned} \quad (8)$$

where the s–channel Z–boson, and the t– and u–channel slepton propagators are given by

$$\begin{aligned}
D_Z &= \frac{m^2_{\tilde{\chi}^0_i}}{s - m^2_Z + i m_Z \Gamma_Z}\\
D_{tL,R} &= \frac{m^2_{\tilde{\chi}^0_i}}{t - m^2_{\tilde{e}_{L,R}} + i m_{\tilde{e}_{L,R}}\Gamma_{\tilde{e}_{L,R}}}\\
D_{uL,R} &= \frac{m^2_{\tilde{\chi}^0_i}}{u - m^2_{\tilde{e}_{L,R}} + i m_{\tilde{e}_{L,R}}\Gamma_{\tilde{e}_{L,R}}}
\end{aligned} \quad (9)$$

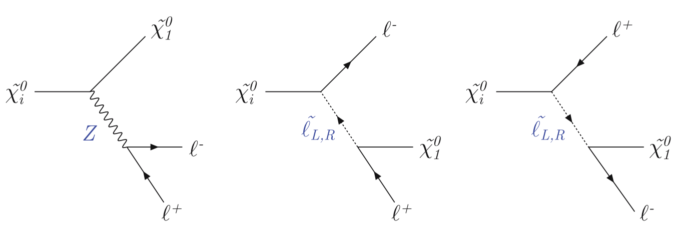

Fig. 1. Diagrams contributing to the leptonic three-body neutralino decay $\tilde{\chi}^0_i \to \tilde{\chi}^0_1 \ell^+\ell^-$; the exchange of the neutral Higgs bosons is neglected because the contribution is strongly suppressed by the tiny electron and muon Yukawa couplings

in terms of the Mandelstam variables, $s = (q_- + q_+)^2$, $t = (p_i - q_+)^2$, and $u = (p_i - q_-)^2$, where p_i, q_+ and q_- are the 4–momenta of the decaying neutralino $\tilde{\chi}^0_i$ and the positively and negatively charged leptons $\ell^\pm$, respectively. The couplings $\mathcal{Z}_{ij}$, g_{Lij} and g_{Rij} are given in terms of the neutralino diagonalization matrix elements $N_{i\alpha}$ $(i,\alpha = 1-4)$:

$$\begin{aligned}
\mathcal{Z}_{ij} &= \frac{1}{2}\left(N_{i3}N^*_{j3} - N_{i4}N^*_{j4}\right)\,,\\
g_{Lij} &= \frac{1}{4 s^2_W c^2_W}(N_{i2}c_W + N_{i1}s_W)(N^*_{j2}c_W + N^*_{j1}s_W)\\
g_{Rij} &= \frac{1}{c^2_W}N_{i1}N^*_{j1}
\end{aligned} \quad (10)$$

The complex couplings satisfy the hermiticity relations:

$$\mathcal{Z}_{ij} = \mathcal{Z}^*_{ji}, \quad g_{Lij} = g^*_{Lji}, \quad g_{Rij} = g^*_{Rji} \quad (11)$$

so that, if the Z–boson and selectron widths are neglected in the Z and selectron propagators, all the bilinear charges $D_{\alpha\beta}$ also satisfy similar relations. In the CP invariant case the mixing matrix elements $N_{i\alpha}$ are purely real or purely imaginary implying that the couplings $\mathcal{Z}_{ij}$, g_{Lij} and g_{Rij} are also purely real or purely imaginary. However, these couplings are in general complex in the CP non-invariant case, having both non–trivial real and imaginary parts.

3.2 Neutralino decay distribution

The absolute amplitude squared of the three–body leptonic decay $\tilde{\chi}^0_i \to \tilde{\chi}^0_1 \ell^+\ell^-$ of a neutralino $\tilde{\chi}^0_i$ with its polarization vector n is given by [18] (the full spin-density decay matrix can be found in Refs. [9, 22])

$$\begin{aligned}
|\mathcal{D}|^2(n) &= 4\,(m^2_i - t)\,(t - m^2_1)(N_1 - N_3)\\
&+ 4\,(m^2_i - u)(u - m^2_1)(N_1 + N_3)\\
&- 8\,m_i m_1\, s\, N_2 + 16\, m_1 \langle p_i n q_- q_+\rangle\, N_4\\
&+ 8\,(n\cdot q_+)\left[m_i(u - m^2_1)(N'_1 + N'_3) - m_1(m^2_i - t)N'_2\right]\\
&+ 8\,(n\cdot q_-)\left[m_i(t - m^2_1)\,(N'_1 - N'_3) + m_1(m^2_i - u)N'_2\right]
\end{aligned} \quad (12)$$

where n is the $\tilde{\chi}^0_i$ spin 4-vector and $\langle p_i n q_- q_+\rangle \equiv \epsilon_{\mu\nu\rho\sigma}p^\mu_i n^\nu q^\rho_- q^\sigma_+$ with the convention $\epsilon_{0123} = +1$. For the sake of notation, we introduce the abbreviations, $m_i = m_{\tilde{\chi}^0_i}$. The seven quartic charges $N_{1,2,3,4}$ and $N'_{1,2,3}$ for the 3–body neutralino decays are defined in terms of the bilinear charges by

$$\begin{aligned}
N_1 &= \frac{1}{4}\left[|D_{RR}|^2 + |D_{LL}|^2 + |D_{RL}|^2 + |D_{LR}|^2\right]\\
N_2 &= \frac{1}{2}\,\mathfrak{Re}\,(D_{RR}D^*_{LR} + D_{LL}D^*_{RL})\\
N_3 &= \frac{1}{4}\left[|D_{LL}|^2 + |D_{RR}|^2 - |D_{RL}|^2 - |D_{LR}|^2\right]\\
N_4 &= \frac{1}{2}\,\mathfrak{Im}\,(D_{RR}D^*_{LR} + D_{LL}D^*_{RL})\\
N'_1 &= \frac{1}{4}\left[|D_{RR}|^2 + |D_{RL}|^2 - |D_{LR}|^2 - |D_{LL}|^2\right]
\end{aligned}$$

$$N_2' = \frac{1}{2}\,\Re\mathfrak{e}\,(D_{RR}D^*_{LR} - D_{LL}D^*_{RL})$$
$$N_3' = \frac{1}{4}\left[|D_{RR}|^2 + |D_{LR}|^2 - |D_{RL}|^2 - |D_{LL}|^2\right] \quad (13)$$

The quartic charges $N_{1,2,3,4}$ are P–even, while the quartic charges $N'_{1,2,3}$ are P–odd.

We choose the rest frame of the decaying neutralino as a reference frame to describe the 3-momenta of the decay products. The neutralino spin 3-vector $\hat{n} = (0,0,1)$ defines the direction of the z-axis. Since the azimuthal angle is irrelevant, the 3-momentum vector of the negative lepton can be taken to fix the x-z plane and its polar angle is denoted by θ $(0 \le \theta \le \pi)$. The orientation of the neutralino decay plane (NDP) is then fully determined by specifying an additional angle α $(0 \le \alpha \le 2\pi)$, so that by rotating the NDP by $-\alpha$ around the ℓ^- direction it is brought to x-z plane, as depicted in Fig. 2. The differential decay distribution is written in terms of two dimensionless energy variables, $x_- = 2E_{e^-}/m_i$ and $x_+ = 2E_{e^+}/m_i$, and the two angles, θ and α, as

$$\frac{d^4\Gamma}{dx_- dx_+ d\cos\theta\, d\alpha} = \frac{\alpha^2 m_i}{16\pi^2} \times \Big[F_0(x_-,x_+) + (\hat{q}_-\cdot\hat{n})\,F_1(x_-,x_+) + (\hat{q}_+\cdot\hat{n})\,F_2(x_-,x_+) + \hat{n}\cdot(\hat{q}_-\times\hat{q}_+)\; F_3(x_-,x_+)\Big] \quad (14)$$

where $\cos\theta = \hat{q}_-\cdot\hat{n}$, $\hat{q}_\pm = \overrightarrow{q}_\pm/|\overrightarrow{q}_\pm|$ and lepton masses are neglected. The four kinematic functions $F_k(x_-,x_+)$ $(k = 0$–$3)$ are expressed in terms of the dimensionless energy variables, x_- and x_+, and the quartic charges as

$$\begin{aligned} F_0(x_-,x_+) &= x_-y_-\,(N_1 - N_3) + x_+y_+\,(N_1+N_3) \\ &\quad - 2r_{i1}(x_- + x_+ - 1 + r_{i1}^2)\,N_2 \\ F_1(x_-,x_+) &= -x_-y_-\,(N_1' - N_3') - r_{i1}x_-x_+\,N_2' \\ F_2(x_-,x_+) &= -x_+y_+\,(N_1'+N_3') + r_{i1}x_-x_+\,N_2' \\ F_3(x_-,x_+) &= r_{i1}\,x_-x_+\,N_4 \end{aligned} \quad (15)$$

where $y_\pm = 1 - x_\pm - r_{i1}^2$ with $r_{i1} = m_1/m_i$. All quartic charges are functions only of the energy variables $x_\pm$, but independent of the orientation angles, θ and α.

In the kinematic configuration of Fig. 2 with $\hat{n} = (0,0,1)$ taken along the positive z–axis and the negative lepton momentum on the x–z plane, the spin dependent scalar products are expressed in terms of the opening angle, χ, and two angles, θ and α, as

$$\begin{aligned} \hat{q}_-\cdot\hat{n} &= \cos\theta, \\ \hat{q}_+\cdot\hat{n} &= \cos\chi\cos\theta - \sin\chi\sin\theta\cos\alpha \\ \hat{n}\cdot(\hat{q}_-\times\hat{q}_+) &= \sin\chi\sin\theta\sin\alpha \end{aligned} \quad (16)$$

where the cosine of two lepton momentum directions, $\cos\chi$, is simply a function of two normalized energy variables, x_- and x_+:

$$\cos\chi = 1 - 2\;(x_- + x_+ - 1 + r_{i1}^2)/(x_-x_+) \quad (17)$$

and $\sin\chi = \sqrt{1-\cos^2\chi}$. The kinematically–allowed ranges for the angles, θ and α, and the dimensionless energy variables, x_+ and x_-, are

$$0 \le \theta,\ \frac{\alpha}{2} \le \pi,\ \ 0 \le x_\pm \le 1 - r_{i1}^2,\ \ (1-x_-)(1-x_+) \ge r_{i1}^2, \quad x_- + x_+ \ge 1 - r_{i1}^2 \quad (18)$$

The allowed crescent–shaped x_+ and x_- region (see the left panel of Fig. 3) is restricted due to four–momentum conservation, while the full ranges of the angles θ and α allowed.

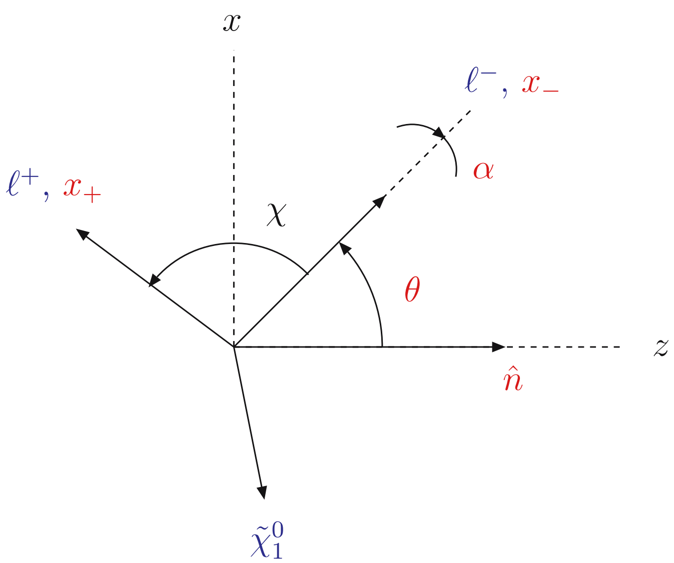

Fig. 2. A schematic diagram of the kinematic configuration of the momenta and spin vector in the initial neutralino rest mass frame for the three-body neutralino decay $\tilde{\chi}^0_i \to \tilde{\chi}^0_1\ell^+\ell^-$ in terms of the two dimensionless energy variables $x_\pm$ and the two angles θ and α. The opening angle χ is uniquely determined by the two variables $x_\pm$

3.3 Majorana nature and implications of CP transformation

Before presenting a few concrete numerical examples for probing the Majorana nature and CP violation in the neutralino system through the leptonic three–body neutralino decays, we investigate the implications of the invariance under CP and CP$\tilde{\text{T}}$ transformations[2] for the three body leptonic neutralino decays [4, 5].

In the rest frame of the decaying neutralino, three final particles form a decay plane and every three–momentum changes its sign under P transformation as well as $\tilde{\text{T}}$ transformation while C transformation exchanges the four momenta of two leptons. The polarization vector $\hat{n}$ does not

[2] The naive time reversal transformation $\tilde{\text{T}}$ reverses the direction of all 3–momenta and spins, but does not exchange initial and final states. Quantities that are odd under CP$\tilde{\text{T}}$ can be non–zero only for complex transition amplitudes with absorptive phases which can be generated, for example, by loops, and Breit–Wigner propagators.

change under P and C transformations, but it changes its sign under $\tilde{\mathrm{T}}$ transformation. Then, the CP operation transforms the momenta and spin vector in the decay processes as

$$x_\pm \to +x_\mp, \quad \vec{q}_\pm \to -\vec{q}_\mp, \quad \hat{q}_\pm \cdot \hat{n} \to -\hat{q}_\mp \cdot \hat{n}, \\ \hat{n}\cdot(\hat{q}_- \times \hat{q}_+) \to -\hat{n}\cdot(\hat{q}_- \times \hat{q}_+) \tag{19}$$

while the CP$\tilde{\mathrm{T}}$ operations transform the momentum and spin vectors as

$$x_\pm \to +x_\mp, \quad \vec{q}_\pm \to +\vec{q}_\mp, \quad \hat{q}_\pm \cdot \hat{n} \to -\hat{q}_\mp \cdot \hat{n}, \\ \hat{n}\cdot(\hat{q}_- \times \hat{q}_+) \to +\hat{n}\cdot(\hat{q}_- \times \hat{q}_+) \tag{20}$$

with the condition that the complex conjugation of the decay amplitude is taken [23].

Since neutralinos are the Majorana particles, CP invariance in the three–body neutralino decay $\tilde{\chi}^0_i \to \tilde{\chi}^0_j \ell^+\ell^-$ leads to additional relations among the bilinear charges defined in (8):

$$D_{LR} = \eta_i\eta_j D_{RR}(t \leftrightarrow u) \\ D_{RL} = \eta_i\eta_j D_{LL}(t \leftrightarrow u) \tag{21}$$

where $\eta_{i,j} = \pm i$ are the intrinsic CP parities [24] of the neutralinos, $\tilde{\chi}^0_{i,j}$, respectively, and, as a result, to the CP relations for the kinematic functions defined in (15):

$$F_0(x_-, x_+) = +F_0(x_+, x_-) \\ F_1(x_-, x_+) = -F_2(x_+, x_-) \\ F_3(x_-, x_+) = -F_3(x_+, x_-) \tag{22}$$

in the three–body leptonic neutralino decays. We note that the CP relations (22) can be satisfied only when the couplings $\mathcal{Z}_{ij}$, g_{Lij} and g_{Rij} are *simultaneously* purely real (for $\eta_i = \eta_j = \pm i$) or *simultaneously* purely imaginary (for $\eta_i = -\eta_j = \pm i$).

On the other hand, CP$\tilde{\mathrm{T}}$ invariance leads to the relations among the bilinear charges:

$$D_{LR} = -D^*_{RR}(t \leftrightarrow u) \\ D_{RL} = -D^*_{LL}(t \leftrightarrow u) \tag{23}$$

These CP$\tilde{\mathrm{T}}$ relations are satisfied if the Z–boson and slepton widths are neglected, that is to say, if there are no absorptive parts in the process. In the approximation of neglecting particle widths, we have the following CP$\tilde{\mathrm{T}}$ relations for the kinematic functions:

$$F_0(x_-, x_+) = +F_0(x_+, x_-) \\ F_1(x_-, x_+) = -F_2(x_+, x_-) \\ F_3(x_-, x_+) = +F_3(x_+, x_-) \tag{24}$$

independently of the mixing character of neutralinos and whether CP is violated or not.

CP–conserving absorptive parts appear in loop diagrams with on–shell propagators through final–state interactions or from the widths of intermediate unstable particles. In addition, CP$\tilde{\mathrm{T}}$–odd asymmetries may arise from the interference between a dominant tree–level and a sub–leading loop diagram mediating the decay. However, those absorptive parts and interference effects are usually tiny in the leptonic decay involving only electroweak interactions, at most at a level of a couple of percents. In this light, we will ignore all the width effects and electroweak loop corrections in the following analytic and numerical analyses.

4 Numerical analyses

In the numerical analyses below we adopt an mSUGRA scenario defined by

$$m_0 = 150\,\mathrm{GeV}, \quad m_{1/2} = 200\,\mathrm{GeV}, \quad A_0 = -650\,\mathrm{GeV} \tag{25}$$

at the GUT scale requiring the pole mass of the top quark $m_t = 178$ GeV, and

$$\tan\beta = 10, \qquad \mathrm{sgn}(\mu) > 0 \tag{26}$$

at the electroweak scale at which all parameters are derived with the RGE code SPheno [25] (very similar results are obtained with other RGE codes; for comparison of different codes see [26]). For the light neutralino and chargino masses we find

$$m_{\tilde{\chi}^0_1} = 78.1\,\mathrm{GeV}, \quad m_{\tilde{\chi}^0_2} = 148.5\,\mathrm{GeV}, \quad m_{\tilde{\chi}^\pm_1} = 148.4\,\mathrm{GeV} \tag{27}$$

and we note that the light neutralino and chargino masses and mixing angles are reproduced fairly well with the tree-level formulae taking

$$M_1 = 80\,\mathrm{GeV}, \;\; M_2 = 158\,\mathrm{GeV}, \;\; |\mu| = 415\,\mathrm{GeV}, \;\; \Phi_\mu = 0; \\ \tan\beta = 10 \tag{28}$$

The selectron and sneutrino masses derived with the RGE code SPheno are

$$m_{\tilde{e}_L} = 207.7\,\mathrm{GeV}, \; m_{\tilde{e}_R} = 173.1\,\mathrm{GeV}, \; m_{\tilde{\nu}_e} = 192.1\,\mathrm{GeV} \tag{29}$$

Relevant for our analysis, the derived branching ratios for leptonic three–body decays of the second lightest neutralino are

$$\mathrm{Br}(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 e^+e^-) = 4.5\%, \qquad \mathrm{Br}(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \mu^+\mu^-) = 4.6\% \tag{30}$$

and for the $\tilde{e}_L \to \tilde{\chi}^0_2 e$ decay

$$\mathrm{Br}(\tilde{e}_L \to \tilde{\chi}^0_2 e) = 28.4\% \tag{31}$$

while other decay modes have $\mathrm{Br}(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \tau^+\tau^-) = 58.9\%$, $\mathrm{Br}(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 q\bar{q}) = 9.1\%$, $\mathrm{Br}(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \nu\bar{\nu}) = 23.7\%$, $\mathrm{Br}(\tilde{e}_L \to \tilde{\chi}^0_1 e) = 21.4\%$, $\mathrm{Br}(\tilde{e}_L \to \tilde{\chi}^-_1 \nu) = 50.3\%$, $\mathrm{Br}(\tilde{\nu}_e \to \tilde{\chi}^0_2 \nu) = 19.6\%$, $\mathrm{Br}(\tilde{\nu}_e \to \tilde{\chi}^+_1 e) = 45\%$, and $\mathrm{Br}(\tilde{t}_1 \to b\tilde{\chi}^+_1) = 98.3\%$, $\mathrm{Br}(\tilde{t}_1 \to c\tilde{\chi}^0_2) = 1.6\%$.

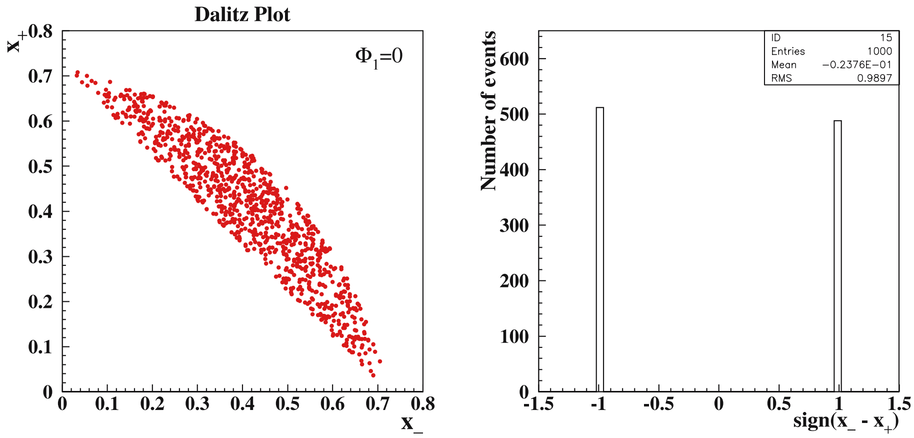

Fig. 3. Left: the Dalitz plot of the neutralino decay $\tilde\chi^0_2 \to \tilde\chi^0_1 \ell^+\ell^-$ in the (x_-, x_+) Dalitz plane. *Right*: the number of events with $\mathrm{sign}(x_- - x_+) = -$ (*left histogram*) and $\mathrm{sign}(x_- - x_+) = +$ (*right histogram*). $\Phi_1 = 0$ is taken and the parameter set (28) is used for the other relevant SUSY parameters. The event distribution is generated with the total number of events of 1000 by a Monte Carlo method

The production cross sections for selectron-pair production processes with unpolarized e^+e^- beams at $\sqrt{s} = 500$ GeV are as follows

$$\sigma\{\tilde e^+_R \tilde e^-_R\} = 273.4\,\mathrm{fb}, \quad \sigma\{\tilde e^\pm_R \tilde e^\mp_L\} = 113.5\,\mathrm{fb}, \\ \sigma\{\tilde e^+_L \tilde e^-_L\} = 80.7\,\mathrm{fb} \tag{32}$$

so that large ensembles of events, $\sim 2\times 10^5$ events for $\tilde e^\pm_R \tilde e^\mp_L$ and $\tilde e^+_L \tilde e^-_L$ at integrated luminosity of 1000 fb^{-1}, will be generated. Given the branching fractions in (30) and (31), a sufficient number of events for the decays $\tilde\chi^0_2 \to \tilde\chi^0_1 e^+e^-$ and $\tilde\chi^0_2 \to \tilde\chi^0_1 \mu^+\mu^-$ are expected to be selected[3], allowing the analysis of the properties of the neutralino decay at great detail.

For completeness, we note that other production channels for SUSY-related processes that are either open or with cross sections greater than 1 fb at $\sqrt{s} = 500$ GeV have the following cross sections: $\sigma\{\tilde\chi^0_1\tilde\chi^0_1\} = 264.8$ fb, $\sigma\{\tilde\chi^0_1\tilde\chi^0_2\} = 159.1$ fb, $\sigma\{\tilde\chi^0_2\tilde\chi^0_2\} = 116.8$ fb, $\sigma\{\tilde\chi^+_1\tilde\chi^-_1\} = 294$ fb, $\sigma\{\tilde\mu^+_R\tilde\mu^-_R\} = 39.8$ fb, $\sigma\{\tilde\mu^+_L\tilde\mu^-_L\} = 22.3$ fb, $\sigma\{\tilde\tau^+_1\tilde\tau^-_1\} = 52.4$ fb, $\sigma\{\tilde\tau^\pm_1\tilde\tau^\mp_2\} = 5.2$ fb, $\sigma\{\tilde\tau^+_2\tilde\tau^-_2\} = 17.1$ fb, $\sigma\{\tilde\nu_e\tilde\nu^*_e\} = 662.2$ fb, $\sigma\{\tilde\nu_\mu\tilde\nu^*_\mu\} = 15.2$ fb, $\sigma\{\tilde\nu_\tau\tilde\nu^*_\tau\} = 16.8$ fb, $\sigma\{\tilde t_1 \tilde t^*_1\} = 45$ fb and $\sigma\{h^0 Z\} = 64.6$ fb. Most of these processes can be separated by simple kinematical cuts. Note that the $\tilde\nu_e\tilde\nu^*_e$ production process might be exploited for our purposes if the decay mode $\tilde\chi^+_1 e$ of $\tilde\nu_e$ in one hemisphere could be used to tag the decay mode $\tilde\chi^0_2\nu$ of the second electron–sneutrino in the other hemisphere. However, since the detailed experimental simulations to assess this possibility are beyond the scope of the paper, we do not include the $\tilde\nu_e\tilde\nu^*_e$ channel in our analyses.

4.1 Lepton energy distribution

The polarization–independent kinematic function F_0 is symmetric with respect to the energy variables x_+ and x_- exactly in the CP invariant case and to a good approximation in the CP non–invariant case. One of the most decisive ways for confirming the Majorana nature of the neutralinos, namely the observation of the symmetric distribution of events on the (x_-, x_+) Dalitz plane [4], can therefore be realized in the cascade decay process of (2) in which the $\tilde\chi^0_2$ rest frame is reconstructable.

The left panel of Fig. 3 shows the Dalitz plot of the leptonic three–body decay $\tilde\chi^0_2 \to \tilde\chi^0_1 \ell^+\ell^-$ in the (x_-, x_+) plane for the parameter set (28) with $\Phi_1 = 0$. The right panel shows the numbers of piled–up events with $\mathrm{sign}(x_- - x_+) = -$ (left histogram) and $\mathrm{sign}(x_- - x_+) = +$ (right histogram), simulated with 1000 events by a Monte Carlo method. We note that the difference of two numbers of events $\Delta N_{\mathrm{ev}} \approx 24$ is within the expected statistical error of $\Delta N_{\mathrm{exp}} = \sqrt{N_{\mathrm{ev}}} \simeq 32$ with $N_{\mathrm{ev}} = 1000$.

4.2 Lepton angular distribution

Another kinematic distribution of great interest is the lepton angle distribution with respect to the neutralino polarization vector; the lepton angle distribution with respect to the beam direction in the e^+e^- reference frame has been discussed in [5]. Defining $\theta_\pm$ to be the polar angle between the $\ell^\pm$ momentum and polarization vector $\hat n$, the normalized lepton angle distribution can be written as

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\pm} = \frac{1}{2}\left(1 \pm \xi_\pm \cos\theta_\pm\right) \tag{33}$$

with $\cos\theta_\pm \equiv \hat q_\pm \cdot \hat n$. In the approximation of all particle widths neglected, the slope parameters have to be equal, $\xi_- = \xi_+$, *irrespective of whether the theory is CP invariant or not*. This is the consequence of CP$\tilde{\mathrm{T}}$ invariance of the decay distribution of the Majorana particle, cf. the second relation of (24). As a result, the sum of two lepton angle distributions has to be flat, that is to say, independent of the polar angles. This is one of the genuine tests of the Majorana nature of the neutralinos.

The left panel of Fig. 4 shows the lepton angle distribution for the parameter set (28) with the phase $\Phi_1 = 0$. The solid line is for the cosine of the negatively–charged lepton angle, $\cos\theta_-$, and the dashed line for the cosine of the positively–charged lepton angle, $\cos\theta_+$. A simple numerical analysis based on the number of events of $N_{\mathrm{ev}} = 1000$ shows that the CP$\tilde{\mathrm{T}}$ relation can be confirmed within 1–σ

[3] In Monte Carlo simulations we conservatively assume that at least 1000 neutralino decay events can be selected and we evaluate all the relevant physical quantities with their tree–level formulas based on the parameter set (28).

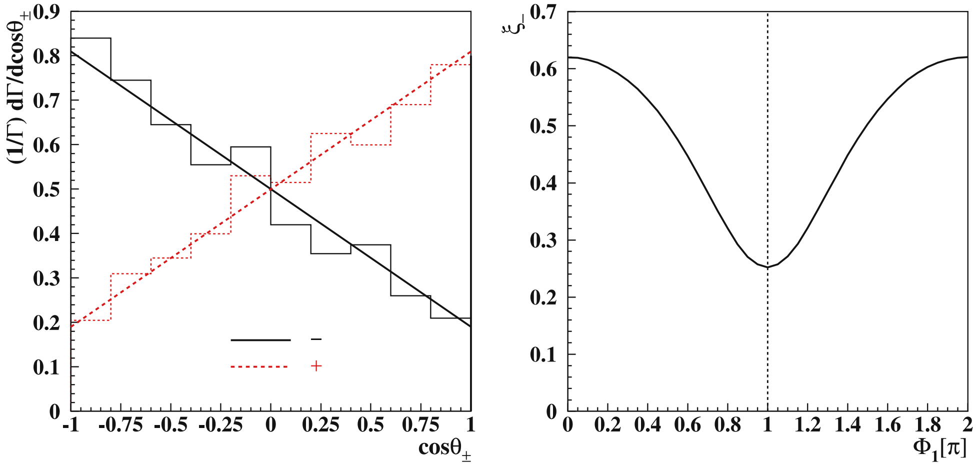

Fig. 4. Left: The normalized lepton angle distribution (33) of the neutralino decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-$; the *black solid line* is for the negative charged lepton and the *red dashed line* for the positive charged lepton. *Right*: The Φ_1 dependence of the slope parameter ξ_- for the parameter set (28)

statistical uncertainty of about 8% for the whole range of $\cos\theta_\pm$. It increases to about 10% for the range with $|\cos\theta_\pm| > 0.8$ vetoed if the cut turns out necessary to avoid distortions of the $\ell^\pm$ distributions differently by experimental selection criteria [16]. Certainly, it will be necessary to perform a more detailed and realistic experimental analysis including all the systematic uncertainties before drawing a more concrete conclusion. Such a comprehensive analysis is, however, beyond the scope of the present theoretical investigations.

The quantities $\xi_\pm$, denoting the slope of the lepton angle distribution with respect to $\cos\theta_\pm$, depend on the values of the SUSY parameters. In the right panel of Fig. 4 the Φ_1 dependence of the CP–even quantities $\xi_\pm$ is shown demonstrating that the $\xi_\pm$ measurement can provide information on the CP-violating phase.

4.3 Lepton invariant mass and opening–angle distribution

The invariant mass m_{ll} of two final–state leptons in the decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-$ is a Lorentz–invariant kinematic variable so that it is straightforward to reconstruct the quantity experimentally by measuring the four momenta of two final–state leptons in the laboratory frame. Furthermore, the distribution for the invariant mass m_{ll} is independent of the specific production process for the decaying neutralino, because the invariant mass distribution does not involve any spin correlations between production and decay [5].

Near the maximum end point of the lepton invariant mass distribution, the neutralino $\tilde{\chi}^0_1$ is produced nearly at rest, the Mandelstam variable s approaches to $(m_2 - m_1)^2$ and the variables t and u become identical $\sim m_1 m_2$. Ignoring the particle widths in the propagators we find from (7) that the decay amplitude can be written approximately as

$$\mathcal{D} \sim \frac{1}{m_2} \bar{u}(\tilde{\chi}^0_1) \Big\{ \not{L} \left[i\mathfrak{Im}(X_L) + \gamma_5 \mathfrak{Re}(X_L) \right] + \not{R} \left[i\mathfrak{Im}(X_R) + \gamma_5 \mathfrak{Re}(X_R) \right] \Big\} u(\tilde{\chi}^0_2) \quad (34)$$

where $X_L = D_{LL}$ and $X_R = D_{LR}$ near the invariant mass end point and $L^\mu / R^\mu = \bar{u}(\ell^-)\gamma^\mu P_{L,R}\, v(\ell^+)$ are the left–handed/right–handed lepton vector currents. The approximate form of the decay amplitude leads to the absolute amplitude squared of the following approximate form:

$$|\mathcal{D}|^2 \sim r_{21}\,(1 - r_{21})^2 \left\{ \mathfrak{Re}(X_L)^2 + \mathfrak{Re}(X_R)^2 \right\} + O(\beta^2) \quad (35)$$

near the end point with the neutralino $\tilde{\chi}^0_1$ velocity $\beta \sim \sqrt{1-\mu_{ll}}$ for the normalized invariant mass, $\mu_{ll} = m_{ll}/(m_2 - m_1)$. Therefore, the invariant mass distribution exhibits a characteristic steep S-wave (slow P-wave) decrease proportional to β (β^3) when the CP parities of two neutralinos are the same (opposite), that is to say, if $X_{L,R}$ is purely real (purely imaginary). On the other hand, in the CP non-invariant case, where both the real and imaginary parts of $X_{L,R}$ are non–vanishing, the invariant mass distribution decreases steeply in S–wave [6].

The threshold behavior of the invariant mass distribution can be understood by investigating the selection rule of the orbital angular momentum by CP symmetry. In the non–relativistic limit of two neutralinos, the orbital angular momentum L of the final two–lepton and LSP system satisfies the CP relation

$$1 = -\eta_1\eta_2\,(-1)^L \quad (36)$$

With the same (opposite) relative CP parity leading to $\eta_1\eta_2 = -1\,(+1)$, the selection rule (36) forces the final two–lepton and LSP system to have $L = 0$ (1) near the threshold. In other words, the neutralino axial–vector (vector) current corresponds to S–wave (P–wave) excitations.

Additional clear signature of the selection rule (36) is provided by the decay distribution with respect to the opening angle χ between two leptons [5]. Since the relation between the invariant mass and the opening angle is given by

$$m^2_{ll} = \frac{m^2_2}{2}\, x_+ x_-\, (1 - \cos\chi) \quad (37)$$

the invariant mass m_{ll} takes its maximum value for $\chi = \pi$ for given x_+ and x_-, *i.e.* when the momentum directions

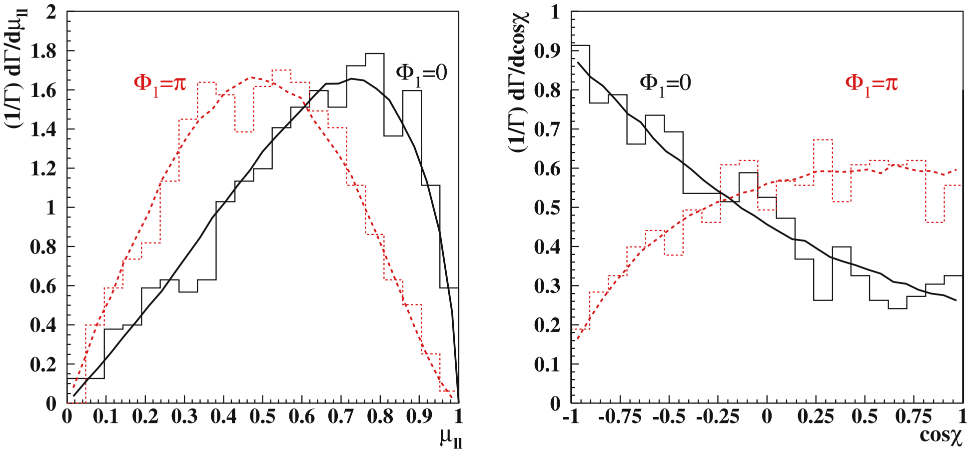

Fig. 5. The lepton normalized invariant mass distribution (*left panel*) and the opening angle distribution (*right panel*) in the three-body leptonic neutralino decay $\tilde\chi^0_2 \to \tilde\chi^0_1 \ell^+\ell^-$. The *black solid lines* are for $\Phi_1 = 0$, (i.e. neutralinos with the same CP parities) and the *red dashed lines* for $\Phi_1 = \pi$, (i.e. neutralinos with the opposite CP parities)

of two leptons are opposite. In this case, the helicities of two leptons coupled to a vector current are opposite, rendering its total spin sum to be unity along the flight direction. Therefore, angular momentum conservation forces the orbital angular momentum to be zero. As a consequence, when two neutralinos have the same (opposite) CP parity, the opening angle distribution is enhanced (suppressed) near $\cos\chi = -1$. A similar argument based on total angular momentum conservation can be applied to show that the opening angle distribution exhibits the opposite property for $\cos\chi = 1$, i.e. for the vanishing opening angle, $\chi = 0$.

Based on the parameter set (28), we show in Fig. 5 the lepton normalized invariant mass distribution (left panel) and the opening–angle distribution (right panel) for $\Phi_1 = 0$ (black solid line) and $\Phi_1 = \pi$ (red dashed line). The histograms are based on 1000 events generated by Monte Carlo. The invariant mass distribution decreases steeply (slowly) near the end point and the opening angle distribution is strongly enhanced (suppressed) near $\chi = \pi$ for two neutralinos of the same (opposite) CP parities in the CP invariant case. Note however, that since in both cases the invariant mass distribution vanishes at the end point, the distinction between the S- and P-wave behavior might be tricky. On the other hand, the opening angle distribution can have a finite value (depending on the relative CP parity) facilitating the discrimination. In our numerical case, the χ^2/n (with $n = 20$ degrees of freedom) for the fits with $\Phi_1 = 0/\pi$, respectively, are as follows: 0.86/0.64 for the lepton invariant mass distributions (left panel), and 0.83/0.54 for the opening angle distributions (right panel), indicating that the theoretical curves fit the Monte–Carlo generated histograms with very good precision. Consequently, the invariant mass and/or opening angle distributions can provide us with a very powerful handle for determining the relative CP parity of two neutralino states $\tilde\chi^0_1$ and $\tilde\chi^0_2$.

4.4 CP–odd triple spin/momentum product

The CP and CP$\tilde{\rm T}$ relations in (22) and (24) enable us to construct a genuine CP–odd and CP$\tilde{\rm T}$–even distribution[4]:

$$F_{\rm CP}(x_-, x_+) = \frac{1}{2}\left[F_3(x_-, x_+) + F_3(x_+, x_-)\right] \qquad (38)$$

A typical CP–odd observable related to the CP–odd distribution (38) is the triple product

$$O_{\rm CP} = \hat n \cdot (\hat q_+ \times \hat q_-) \qquad (39)$$

formed with the spin vector $\hat n$ and two final lepton momentum directions. With the CP–odd observable we can construct the CP–odd asymmetry as follows:

$$A_{\rm CP} \equiv \frac{N(O_{\rm CP} > 0) - N(O_{\rm CP} < 0)}{N(O_{\rm CP} > 0) + N(O_{\rm CP} < 0)} = \frac{\int_{\mathcal D} \frac{1}{2}\sin\chi\, F_{\rm CP}(x_-, x_+)\, dx_- dx_+}{\int_{\mathcal D} F_0(x_-, x_+)\, dx_- dx_+} \qquad (40)$$

where $\mathcal D$ denotes the kinematically allowed (x_-, x_+) Dalitz region defined in (18).

As described in Sect. 2, CP violation in the neutralino system arises when the phases of M_1 and/or μ are different from 0 and/or π. These phases generally lead to large contributions to the EDMs. For the selectron masses under consideration in the present work, the experimental bounds on the electron EDM put the strongest constraints on the phases Φ_1 and Φ_μ. Nevertheless, it has been demonstrated in a recent work [21] that if the phase Φ_μ is set to be 0 or π, the full range of Φ_1 is allowed. In this light, we set Φ_μ to be zero and vary Φ_1 in estimating the CP–odd asymmetry $A_{\rm CP}$.

Figure 6 shows the Φ_1 dependence of the CP–odd and CP$\tilde{\rm T}$–even asymmetry $A_{\rm CP}$ of the triple scalar product. Numerically we find that the magnitude of the asymmetry can be as large as 15% for the parameter set (28), which is expected to enable us to clearly measure CP violation in the neutralino system directly with its expected 1–σ

[4] In the presence of absorptive CP–conserving phases due to particle widths or radiative corrections, there exist two additional CP–odd distributions; $F_0(x_-, x_+) - F_0(x_+, x_-)$ and $F_1(x_-, x_+) + F_2(x_+, x_-)$.

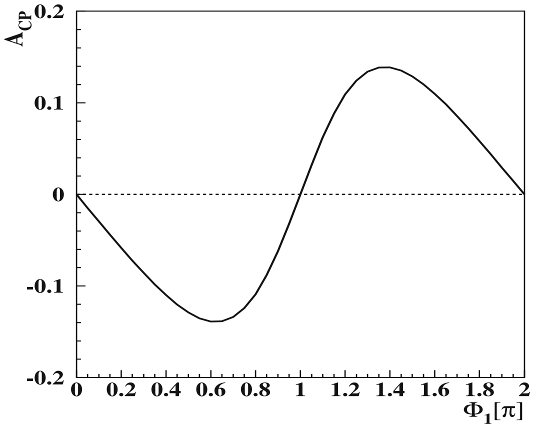

Fig. 6. The Φ_1 dependence of the CP odd and $CP\tilde{T}$ even asymmetry A_{CP} of the triple scalar product for the parameter set (28). We note that the 1σ statistical uncertainty of the CP odd asymmetry is $\sqrt{(1-A_{CP}^2)/N_{\rm ev}} \simeq 3.1\%$ with the number of events of $N_{\rm ev} = 1000$

statistical uncertainty of $\sqrt{(1-A_{\rm CP}^2)/N_{\rm ev}} \simeq 3.1\%$ for the number of events of $N_{\rm ev} = 1000$.

We note finally that if the measured value of $A_{\rm CP}$ turns to be very small, close to zero, the two-fold ambiguity $\Phi_1 = 0$ or $\Phi_1 = \pi$, can be resolved with the help of either the slope parameters $\xi_\pm$ or the lepton invariant mass/opening angle distribution, see Fig. 4 and Fig. 5, respectively.

5 Conclusions

Taking into account the possibility of having highly polarized neutralinos $\tilde{\chi}^0_2$ and of reconstructing their rest frames, we have performed a systematic analysis of the polarized neutralino decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-$ in its decay rest frame for all the physical implications due to the Majorana nature as well as CP violation of the neutralino system in the MSSM. We have demonstrated that the decay process (1) in the cascade channel like (2) can be used to provide alternative powerful methods for probing the neutralino system in detail.

In the CP invariant case, the Majorana nature of the neutralinos can be checked through the leptonic decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-$ by verifying that in the $\tilde{\chi}^0_2$ rest frame:

- the charged lepton energy distribution is identical irrespective of the electric charge of the lepton,
- the sum of the negative and positive lepton angle distributions is independent of the lepton angles with respect to the neutralino polarization vector,
- the relative CP parity of two neutralinos can be identified by measuring the threshold behavior of the invariant mass distribution near the kinematic end point and/or the dependence of the decay distribution on the opening angle near the angle close to π and/or 0.

In the CP non–invariant case, if all the absorptive parts (which are expected to be usually tiny for such an electroweak decay) are ignored, both the lepton energy and angle distributions in the decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-$ can allow us to probe the Majorana nature of the neutralinos as in the CP invariant case. In addition, the three–body leptonic neutralino decays allow us to construct a CP–odd triple scalar product of the neutralino polarization vector and two lepton momenta. Numerically, we have found that for the parameter set (28) the CP asymmetry related to the triple product could be of the order of 15%, while satisfying the severe EDM constraints on the SUSY parameters.

Finally, we emphasize that the analyses in the present work are only statistical and based on the assumption that both the degree of neutralino polarization and the efficiency of reconstructing the neutralino rest frame are 100%. Clearly, it is necessary to perform further detailed experimental analyses with realistic values of reconstruction efficiencies and with background processes included, which is beyond the scope of the present theoretical analyses. However, we think that the results of our theoretical studies are encouraging enough to motivate further detailed experimental investigations to assess the feasibility for probing the Majorana nature of neutralinos and CP violation in the neutralino system at future colliders.

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